Ponderomotive ratchet in a uniform magnetic field

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We show how a ratchet effect, generally used in systems with periodic potentials, can also be practiced on charged particles by an ac field alone, in a background magnetic field near the cyclotron resonance. The effect relies entirely on the spatial inhomogeneity of the high-frequency drive, which produces a deterministic asymmetric ponderomotive barrier for undamped particles. Such a barrier can reflect particles incident from one side while transmitting those incident from the opposite side, hence acting somewhat like a Maxwell demon. The necessary fields are perhaps most easily realized in a plasma, though the effect is more general.

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I. INTRODUCTION

In spatially-periodic structures, the nonlinear dynamics of particles under an external ac drive can give rise to uncompensated particle flow in a preferred direction, even without a biased field. These phenomena, also called ratchet effects, are of particular interest to the understanding of molecular motors in biological systems and have been intensively studied in a variety of models [1–3]. Traditionally, ratchet effects are attributed to periodic potentials, in which particles exhibit Hamiltonian stochastic dynamics [4,5], or dissipative, either chaotic [2,3] or regular [6], dynamics under ac drive. However, a class of ratchets apparently escaped consideration. What this paper shows is that, surprisingly, a ratchet effect can also (i) rely entirely on the spatial asymmetry of a monochromatic ac force applied over a uniform background and, simultaneously, (ii) be practiced on particles undergoing regular Hamiltonian motion in the ac field. Like previously investigated ratchets [3,4,7], the scheme produces a rectification effect in initially thermal ensembles of particles; it also can be utilized for particle confinement, separation, and cooling.

The essence of a ratchet for undamped particles is an asymmetric barrier. Thermal particles traveling through a chain of such barriers are transmitted in one direction with probability $p>\frac{1}{2}$, which is higher than the probability to be pushed in the other direction, $1-p<\frac{1}{2}$; as a result, an average particle flow is generated. Traditionally, it is assumed that a combination of corrugated background and spatially uniform ac field is necessary to drive the effect. However, uniform (or smooth) background fields together with an inhomogeneous ac field are as well suitable for this purpose. The idea can be explained as follows. Under intense ac drive, a particle undergoes fast oscillations superimposed on the average drift motion. If the particle drift displacement on a period of these oscillations is sufficiently small, the average effect of the ac drive can approximately be replaced by particle interaction with so-called ponderomotive potential [8,9]. However, the limitation imposed by this energy-conservation property of the average ponderomotive force can be broken if the particle interaction with the ac field is nonadiabatic. In this case, by adjusting the field structure, this force can be made different for particles incident on different sides of the interaction region; hence the ac barrier can operate as a one-way wall and produce a ratchet effect.

A technique, which effectively amounts to a ponderomotive ratchet, was recently proposed in the optical range of frequencies to achieve atomic cooling effects [10,11]. The suggestions require a bichromatic ac field. As a result, a ponderomotive force employed is essentially phase-dependent. This makes the barrier probabilistic even for given velocity $[\frac{1}{2}<p(\mathbf{v})<1]$, which is also usual for other traditional ratchet-type devices. Strikingly though, a deterministic $[p(\mathbf{v})=1]$ asymmetric barrier for undamped particles undergoing regular motion can also be produced by a ponderomotive force. A particular kind of such barriers was proposed in Refs. [12–14]. It was shown that a deterministic one-way wall can be generated, for selected plasma constituents, by an ac field at frequency ω in an inhomogeneous dc magnetic field near the cyclotron resonance $\omega=\Omega(z)$, if $\omega-\Omega(z)$ changes sign inside the interaction region. Putting aside the unavoidable resonant heating of transiting particles, such a barrier acts essentially like a Maxwell demon and can be employed in various applications, including the selective separation of plasma species [15], confinement of one-component plasmas, enhancement of multiple-mirror plasma confinement [9,16], and current drive [12–14,17].

Remarkably, one can also propose an alternative type of ponderomotive one-way walls, in which the resonance frequency Ω does not have to vary in space, so that a deterministic ratchet can rely *entirely* on the ac field inhomogeneity. The purpose of this paper is to show that an ac drive with a particular spatial profile in the presence of a uniform (or quasi-uniform) dc magnetic field is itself sufficient to generate average flow of particles exhibiting regular Hamiltonian dynamics in the interaction region. The advantage of the technique operating at uniform background is that it is more easily implemented when sustaining a periodic potential together with the ac field might be difficult. In particular, compared to the previous scheme [13,14], such a ratchet does not require sustaining a large ac field amplitude at the cyclotron resonance, which might be impeded by the collective plasma response.

The paper is organized as follows. In Sec. II, we briefly restate the concept of a ponderomotive potential, in particular, for a particle in a dc magnetic field. In Sec. III, we discuss the operation of a single asymmetric ponderomotive barrier in a uniform (or quasi-uniform) dc magnetic field. In Sec. IV, we explain how a ratchet effect can be produced by means of such barriers. In Sec. V, we summarize the main ideas of the paper.

II. ADIABATIC PONDEROMOTIVE POTENTIAL

Consider a charged particle driven by an intense ac field $\mathbf{E}=\mathrm{Re}\,\mathbf{E}_c, \mathbf{E}_c=\mathbf{E}_0\exp(-i\omega t)$, assuming that the characteristic scale L of $\mathbf{E}_0(\mathbf{r})$ is large compared to the amplitude of the particle quiver motion. In this case, the particle can be treated as a dipole located at the average position $\mathbf{R}=\langle\mathbf{r}\rangle$ with a dipole moment $\mathbf{p}$ generally being a functional of $\mathbf{E}[\mathbf{R}(t)]$. The average ponderomotive force on the particle can then be approximately represented as follows:

$$\langle\mathbf{F}\rangle = \langle(\mathbf{p}\cdot\nabla)\mathbf{E}\rangle + \frac{1}{c}\langle\dot{\mathbf{p}}\times\mathbf{B}\rangle. \tag{1}$$

Here $\langle\cdots\rangle$ stands for averaging over the ac period, c is the speed of light, and $\mathbf{B}=\mathrm{Re}\,\mathbf{B}_c$ is the ac magnetic field; both $\mathbf{E}$ and $\mathbf{B}$ are assumed evaluated at location $\mathbf{R}$, and $\mathbf{B}_c=-i(c/\omega)\nabla\times\mathbf{E}_c$. At sufficiently small drift velocity (see below), one can employ the "adiabatic" expression for $\mathbf{p}$, that is, $\mathbf{p}=\mathrm{Re}[\boldsymbol{\alpha}\mathbf{E}_c(\mathbf{R})]$, where $\boldsymbol{\alpha}(\omega)$ is the particle polarizability tensor. In this case, the average force on the particle can be approximately represented as $\mathbf{F}=-\nabla\Phi$, where the so-called ponderomotive potential Φ is given by [18]

$$\Phi = -\tfrac{1}{4}(\mathbf{E}_0^*\cdot\boldsymbol{\alpha}\cdot\mathbf{E}_0). \tag{2}$$

Equation (2) can also be represented in an equivalent form

$$\Phi = \frac{e^2}{4m\omega^2}\sum_\nu \lambda_\nu |E_\nu|^2, \tag{3}$$

where e and m are, respectively, the charge and the mass of the particle; λ_ν are the eigenvalues of a dimensionless tensor $\mathrm{T}=-(m\omega^2/e^2)\boldsymbol{\alpha}$ corresponding to the eigenvectors $\boldsymbol{\tau}_\nu$, and $E_\nu=\mathbf{E}_0\cdot\boldsymbol{\tau}_\nu^*$. For a particle in vacuum, T is a unit tensor, and Eq. (3) yields the well known expression $\Phi=\Phi_\mathrm{v}\equiv(e^2/4m\omega^2)|E_0|^2$ [8,9]. However, in the presence of a dc magnetic field $\mathbf{B}_0=\mathbf{z}^0B_0$, the particle polarizability is modified, so that T can now be expressed as

$$\mathrm{T} = \begin{pmatrix} \frac{1}{1-b^2} & \frac{ib}{1-b^2} & 0 \\ \frac{-ib}{1-b^2} & \frac{1}{1-b^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

where $b=\Omega/\omega$ and $\Omega=eB_0/mc$, and thus

$$\boldsymbol{\tau}_{\pm1} = (\mathbf{x}^0 \pm i\mathbf{y}^0)/\sqrt{2}, \quad \lambda_{\pm1} = (1\pm\Omega/\omega)^{-1},$$

$$\boldsymbol{\tau}_0 = \mathbf{z}^0, \quad \lambda_0 = 1. \tag{5}$$

Note that for a wave with resonant circular polarization $\nu=-1$ (we assume $b>0$), the magnitude of Φ increases in comparison with the vacuum case: $\Phi\sim\lambda\Phi_\mathrm{v}$, where the enhancement factor $\lambda\equiv\lambda_{-1}$ grows infinitely as the cyclotron resonance is approached [8,9,19]. A large magnitude of the resonant ponderomotive potential is useful for plasma confinement, employed, for instance, in magnetic mirror devices [9,20]. The same effect can be used for producing a ratchet effect, if the ac field is supplied with an appropriate spatial profile.

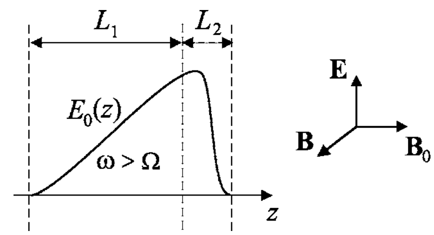

FIG. 1. An ac field profile producing asymmetric ponderomotive barrier in a uniform dc magnetic field: $L_1\gg\Delta l\gg L_2$, where $\Delta l=\lambda v_z/\omega$.

III. ONE-WAY WALL

The ponderomotive one-way wall can be explained as follows. The *true* average force on a particle (1) is proportional to the amplitude of the particle oscillation at the frequency equal or close to ω. Then, to actually "see" the potential Φ, the particle must first gain energy $\mathcal{E}\sim\lambda^2\Phi_\mathrm{v}$, which is $\lambda\gg1$ times larger than the height of the potential:

$$\mathcal{E}\sim\lambda\Phi\gg\Phi. \tag{6}$$

For a particle with kinetic energy less than Φ, to see the barrier will require receiving the energy (6) from the ac field, for which process the characteristic time scale is $\Delta t\sim\lambda/\omega$. If, in a uniform dc magnetic field, the ac field region has a width $L\gg\Delta l, \Delta l=\lambda v_z/\omega$, and the particle longitudinal (drift) and transverse (quiver) energies are less than or comparable with the height of the barrier $\Phi_\mathrm{max}>0$, the particle will be reflected by the barrier, as if $\Phi(\mathbf{r})$ were a true potential. On the other hand, if $L\lesssim\Delta l$, the same particle will be transmitted, as its oscillatory motion will have no time to build up, and hence the repulsive ponderomotive force will have no time to become established.

Consider now a ponderomotive barrier in a plasma with temperature $T\lesssim\Phi_\mathrm{max}$, with ac field having a profile depicted in Fig. 1. Suppose that the left slope of the field has a scale L_1, which is large compared to the characteristic value of Δl for thermal particles. The barrier will then reflect particles incident from the left, preserving both their longitudinal and transverse energies. Suppose now that the right slope has the scale $L_2\ll\Delta l$, so that each particle incident from the right is transmitted through the thin region of repulsive force without substantial energy change. After that, a particle finds itself on the top of the potential hill, from which it further slides off adiabatically, so that the resulting longitudinal and transverse energy changes are given by

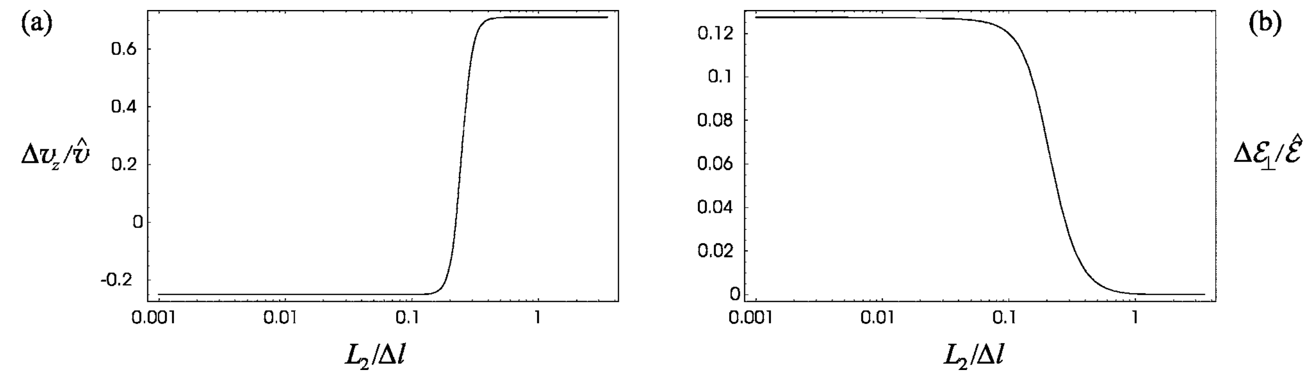

FIG. 2. Longitudinal velocity change Δv_z and transverse energy gain $\Delta\mathcal{E}_\perp$ vs L_2 for a particle after scattering off a barrier depicted in Fig. 1 (results of numerical simulations): $\mathbf{B}_0$ and $\mathbf{E}_0$ given by Eqs. (8); $\Delta l = \lambda v_z/\omega$. The particle initial velocity is $v_{z,0} = -\hat{v}/2\sqrt{2}$; $\hat{v} = (eE_{\max}/m\omega)\sqrt{\lambda} \sim (\Phi_{\max}/m)^{1/2}$; $\hat{\mathcal{E}} = m\hat{v}^2 \sim \lambda\Phi_{\max}$; and $\lambda = 100$. To compare, analytic predictions according to Eq. (7) for $L_2/\Delta l \ll 1$ and $L_2/\Delta l \gg 1$ yield respectively: $\Delta v_z/\hat{v} \approx -0.26, 0.71, \Delta\mathcal{E}_\perp/\hat{\mathcal{E}} \approx 0.125, 0$. Particles incident with the same $|v_{z,0}|$ but from the left are reflected adiabatically, as also checked numerically.

$$\Delta\mathcal{E} = \Phi_{\max}, \quad \Delta\mathcal{E}_\perp = \lambda\Phi_{\max}. \tag{7}$$

The ponderomotive barrier in Fig. 1 is then asymmetric and acts essentially like a Maxwell demon [21], except that it increases the energy of transiting particles, as required by laws of thermodynamics. Note also that the contemplated effect is robust and can be achieved at finite ratio $L_2/\Delta l$ as well. The results of our numerical simulations (Fig. 2) for

$$\mathbf{B}_0(z) = \mathbf{z}^0\left(1 - \frac{1}{\lambda}\right), \tag{8a}$$

$$\mathbf{E}_0(z) = \mathbf{x}^0\frac{a}{2}\exp\left(-\frac{z^2}{L_1^2}\right)\left[1 - \tanh\left(\frac{z}{L_2}\right)\right] \tag{8b}$$

(field amplitudes are measured in units $m\omega c/e$), indicate that the effect persists up to $L_2/\Delta l \sim 1$, whereas at $L_2/\Delta l \gtrsim 1$ the barrier loses the asymmetry. Asymptotic values for $L_2/\Delta l \ll 1$ and $L_2/\Delta l \gg 1$ have also been checked numerically and have been found in agreement with our analytic predictions given in Eq. (7).

To demonstrate the action of a barrier on a particle ensemble, we also calculated a sample velocity mapping produced by this barrier on a thermal distribution with $T = 0.3\Phi_{\max}$ (Fig. 3). As predicted, at these parameters, the majority (about 80%) of the particles incident on the barrier end up on its left side with $v_z < 0$, whereas few (about 20%) particles are reflected by the abrupt slope of the ac field and get $v_z > 0$. The nonvanishing percent of the latter is due to the fact that a thermal distribution always contains particles with sufficiently small initial v_z. Such particles have enough time to develop oscillatory motion even in abrupt fields and actually see the abrupt slope as a repulsive ponderomotive barrier. Some of these particles exhibit deterministic adiabatic reflection, and only those with $v_z \sim \omega L_2/\lambda$ are scattered probabilistically, depending on the phase of the field. Except

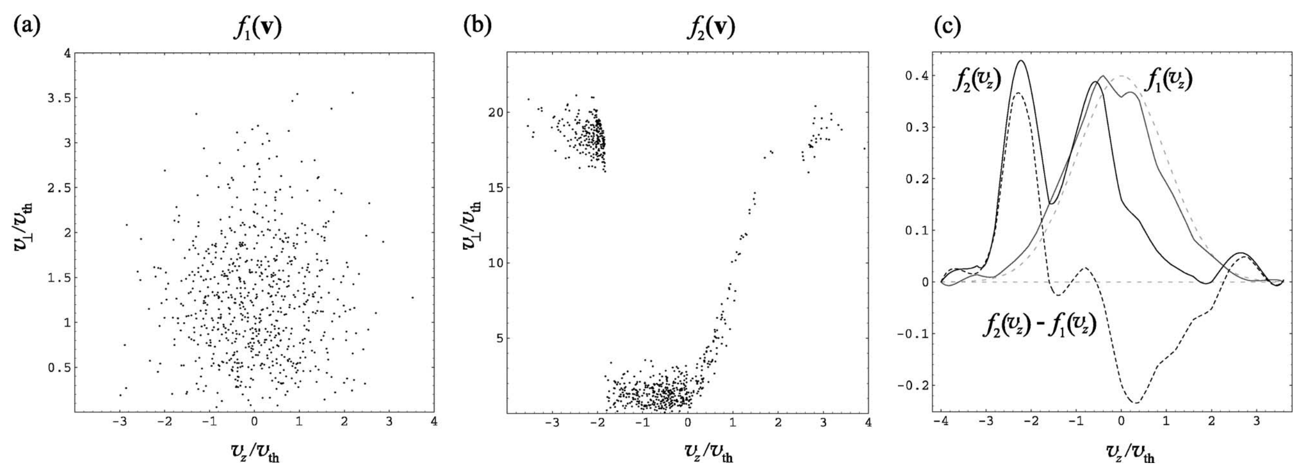

FIG. 3. Velocity mapping produced by a single ponderomotive barrier (8): (a) original near-Gaussian distribution $f_1(\mathbf{v})$; (b) distribution after scattering $f_2(\mathbf{v})$; (c) distribution over longitudinal velocities v_z (dashed is their difference and the exact Gaussian distribution). Velocity is measured in units of thermal velocity $v_{\rm th}$; $\lambda = 100$; $a = 0.001$; $L_1 = 4c/\omega$; $L_2 = 0.02\,\lambda\hat{v}/\omega$; $T = 0.3\Phi_{\max}$; and 720 points used. For most of thermal particles the mapping is deterministic, except for a small fraction of those nonadiabatically scattering off an abrupt slope of the barrier.

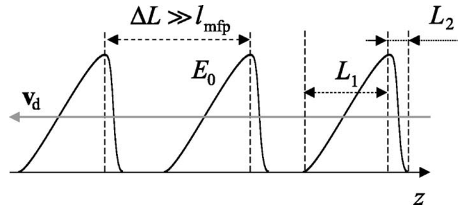

FIG. 4. Chain of asymmetric ponderomotive barriers producing average flow velocity $\mathbf{v}_d$: $L_1 \gg \Delta l \gg L_2, \Delta L \gg l_{\mathrm{mfp}} \gg L_2$; and $\Delta l = \lambda v_z / \omega$.

for the latter, the probability for the barrier to scatter a particle to the left is only a function of velocity: $p = p(\mathbf{v})$, hence the barrier can be considered as deterministic.

IV. PONDEROMOTIVE RATCHET

To see how the barrier can be employed for producing a ratchet effect, consider a chain of barriers, as depicted in Fig. 4. Assume that the distance *between* the barriers ΔL is large compared to the mean free path l_{mfp}, on which the particle energy dissipates. The actual dissipation mechanism is not important. In the simplest case, one can assume collisional decay due to the presence of a background medium, such as gas or plasma; however, the effective dissipation can as well may be due to stochastic interactions with waves, emittance of the cyclotron radiation, etc. The ratio L_1 / l_{mfp} is as well unimportant, but we will require that $L_2 \ll l_{\mathrm{mfp}}$, which provides that each barrier still operates as a one-way wall.

A particle will arrive at each subsequent barrier with $\Delta \mathcal{E}_\parallel$ and $\Delta \mathcal{E}_\perp$ both of the order of T, since the energy gain in Eq. (7) dissipates as the particle travels from one barrier to another. Hence, each next barrier will have the same effect on the particle, which is then propelled by the ac field in a certain direction (Fig. 4). On a distribution of particles, such a field will generate an average particle flow without a bias, entirely due to the asymmetric ponderomotive acceleration, thus producing a ratchet effect. Employing the random-walk approximation and taking $p \approx 1$, the average drift velocity v_{d} can be estimated as

$$v_d \sim v_{\mathrm{th}} \frac{l_{\mathrm{mfp}}}{\Delta L}, \tag{9}$$

where v_{th} is a thermal velocity.

To simulate particle motion in a ponderomotive ratchet, we use a hybrid numerical scheme. For particle interaction with ac field inside an individual barrier, we assume Hamiltonian dynamics and employ precise velocity mapping (Fig. 3) to describe particles scattering off a barrier. This mapping links random-walk trajectories between the barriers, which are calculated by model (Langevin) equations with delta-correlated stochastic force obeying Gaussian statistics. Sample trajectories are shown in Fig. 5 and demonstrate the predicted nonzero drift in $-z$ direction.

If employed particularly for driving a current, such a current source would exhibit the efficiency close to that by barriers in non-uniform magnetic field [13]. However, in contrast to the rf field arrangements considered in Ref. [14], the

FIG. 5. Sample trajectories of particles moving through a ponderomotive ratchet potential depicted in Fig. 4 (arbitrary units). The location of individual barriers is shown by dashed lines ($\Delta L \gg L_1, L_2$).

ratio $\Delta \mathcal{E}_\perp / \Delta \mathcal{E}_\parallel$ (i.e., that of amounts of ac energy spent on particle longitudinal acceleration and transverse heating, respectively) is fixed. In this case, the particle transverse heating cannot be reduced in comparison with longitudinal acceleration. Nevertheless, practicing asymmetric reflection and transmission in a uniform magnetic field could be favorable compared to the previously proposed techniques [13,14], as it does not require precise cyclotron resonance at the maximum ac electric field, and hence is more accessible technologically. Also, it may not be easy, in some cases, to produce magnetic field gradients. Hence, although not more efficient, the constant field ratchets may be more easily realized in practice.

In addition to current drive, the proposed technique can also be used for selective separation of plasma constituents, as the ratchet effect strongly depends on resonant properties of the particles. For example, a noticeably different contribution to the overall ion flow will be provided by different isotopes having slightly different resonant frequencies Ω, if the ac field is tuned to the resonance of one of these species. It is then possible to separate isotopes in space. Related cooling mechanisms also become possible when the height and the location of ponderomotive walls are varied in time [10,22].

Due to the universal nature of resonant ponderomotive interactions, we anticipate that the effects contemplated here are quite general and could be practiced also on neutral particles, possibly, including atoms. If so, these results could supplement the existing techniques of particle manipulation by laser radiation pressure, which broke important ground in atomic physics [23].

V. CONCLUSIONS

In summary, we demonstrated that a ratchet effect can be practiced on particles exhibiting regular Hamiltonian dynamics through the interaction with a high-frequency field. An asymmetric ponderomotive barrier can be produced by inhomogeneous ac drive, in a uniform dc magnetic field near the cyclotron resonance for selected charged-particle constituents. Such a barrier can reflect particles incident from one side while transmitting those incident from the opposite side, hence acting somewhat like a Maxwell demon. Unlike the methods contemplated in Refs. [13,14] for the case of essentially nonuniform magnetic field, the proposed technique is technologically more accessible, as it does not require precise cyclotron resonance at the maximum ac electric field, nor does it require magnetic field gradients.

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